

- 1- Power Flow Analysis.
- 2- Balanced (Symmetrical) Fault Analysis. تحليل العطل المتوازن (المتناظر) في منظومات القدرة الكهربائية
- 3- Symmetrical Components.
- 4- Unbalanced (Unsymmetrical) Fault Analysis.
- 5- Earthing Systems.

## THE POWER FLOW PROBLEM AND ITS SOLUTION

The power flow problem consists of a given transmission network where all lines are represented by a Pi-equivalent circuit and transformers by an ideal voltage transformer in series with an impedance. Generators and loads represent the boundary conditions of the solution. Generator or load real and reactive power involves products of voltage and current. Mathematically, the power flow requires a solution of a system of simultaneous nonlinear equations.

### 1.1 The Power Flow Problem on a Direct Current Network

The problems involved in solving a power flow can be illustrated by the use of direct current (DC) circuit examples. The circuit shown in Figure 4.1 has a resistance of  $0.25 \Omega$  tied to a constant voltage of  $1.0 \text{ V}$  (called the *reference voltage*). We wish to find the voltage at bus 2 that results in a net inflow of  $1.2 \text{ W}$ . Buses are electrical nodes. Power is said to be “injected” into a network; therefore, loads are simply negative injections.

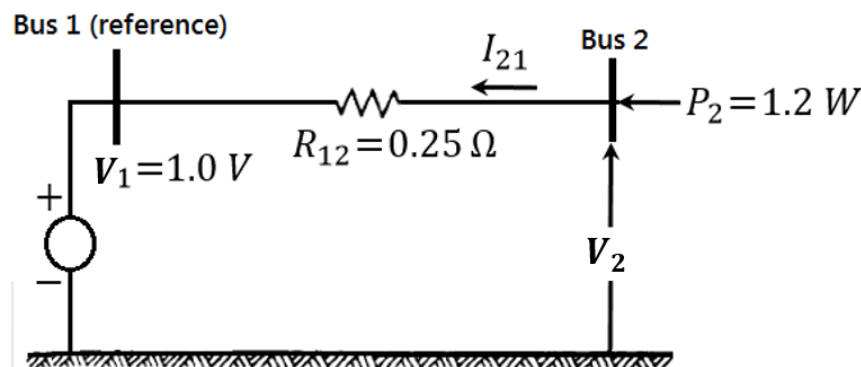


FIG. 1-1 Two-bus DC network.

The current from bus 2 to bus 1 is

$$I_{21} = (V_2 - 1.0) \times \frac{1}{0.25}$$

$I_{21} = (V_2 - 1.0) \times 4$	1-1
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Power  $P_2$  is

$P_2 = 1.2 = V_2 I_{21} = V_2 (V_2 - 1.0) \times 4$	1-2
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or

$4V_2^2 - 4V_2 - 1.2 = 0$	1-3
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The solutions to this quadratic equation are  $V_2 = 1.24162 V$  and  $V_2 = -0.24162 V$ . Note that 1.2 W enter bus 2, producing a current of  $0.96648 A$  ( $V_2 = 1.24162 V$ ), which means that  $0.96648 W$  enter the reference bus and  $0.23352 W$  are consumed in the  $0.25 \Omega$  resistor.

$$V_2 = 1.24162$$

$$V_2 = -0.24162$$

$$I_{21} = \frac{V_2 - V_1}{0.25}$$

$$P_{Loss.} = I^2 R = (0.966)^2 \times 0.25 = 0.2335 W$$

$$\text{And } P_2 - P_{Loss.} = 0.9664 W$$

Let us complicate the problem by adding a third bus and two more lines (see Figure 1.2).

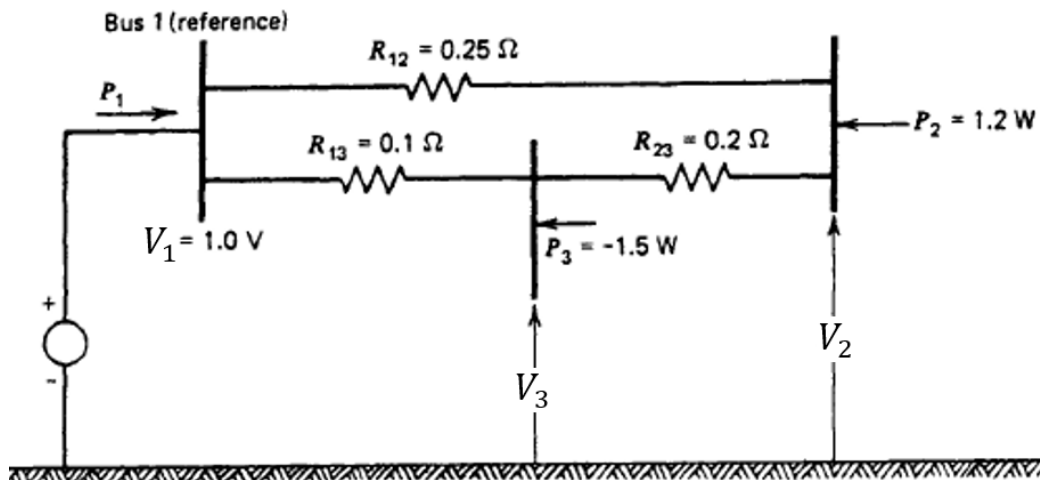


FIG. 1.2 Three-bus DC network

The problem is more complicated because we cannot simply write out the solutions using a quadratic formula. The admittance equations are

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 14 & -4 & -10 \\ -4 & 9 & -5 \\ -10 & -5 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$	1-4
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In this case, we know the power injected at buses 2 and 3 and we know the voltage at bus 1. To solve for the unknowns ( $E_2$ ,  $E_3$  and  $P_1$ ), we write Eqs. 4.5, 4.6, and 4.7.

The solution procedure is known as the **Gauss-Seidel procedure**, wherein a calculation for a new voltage at each bus is made, based on the most recently calculated voltages at all neighboring buses.

$I_2 = \frac{P_2}{V_2} = -4 (1.0) + 9 V_2 - 5 V_3$	
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Bus 2:	$V_2^{new} = \frac{1}{9} \left( \frac{1.2}{V_2^{old}} + 4 + 5V_3^{old} \right)$	1-5
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Where  $V_2^{old}$  and  $V_3^{old}$  are the initial values for  $V_2$ , and  $V_3$ , respectively.

Bus 3:	$I_3 = \frac{P_2}{V_2} = -10(1.0) - 5 V_2^{new} + 15 V_3$	
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Bus 3:	$V_3^{new} = \frac{1}{15} \left[ \frac{-1.5}{V_3^{old}} + 10 + 5 V_2^{new} \right]$	1-6
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where  $V_2^{new}$  is the voltage found in solving Eq. 1-5, and of  $V_3^{old}$  is the initial value of  $V_3$ .

Bus 1:	$P_1 = V_1 I_1^{new} = 1.0 I_1^{new} = 14 - 4 V_2^{new} - 10 V_3^{new}$	1-7
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The Gauss-Seidel method first assumes a set of voltages at buses 2 and 3 and then uses Eqs. 1-5 and 1-6 to solve for new voltages. The new voltages are compared to the voltage's most recent values, and the process continues until the change in voltage is very small. This is illustrated in the flowchart in Figure1-3 and in Eqs. 1-8 and 1-9.

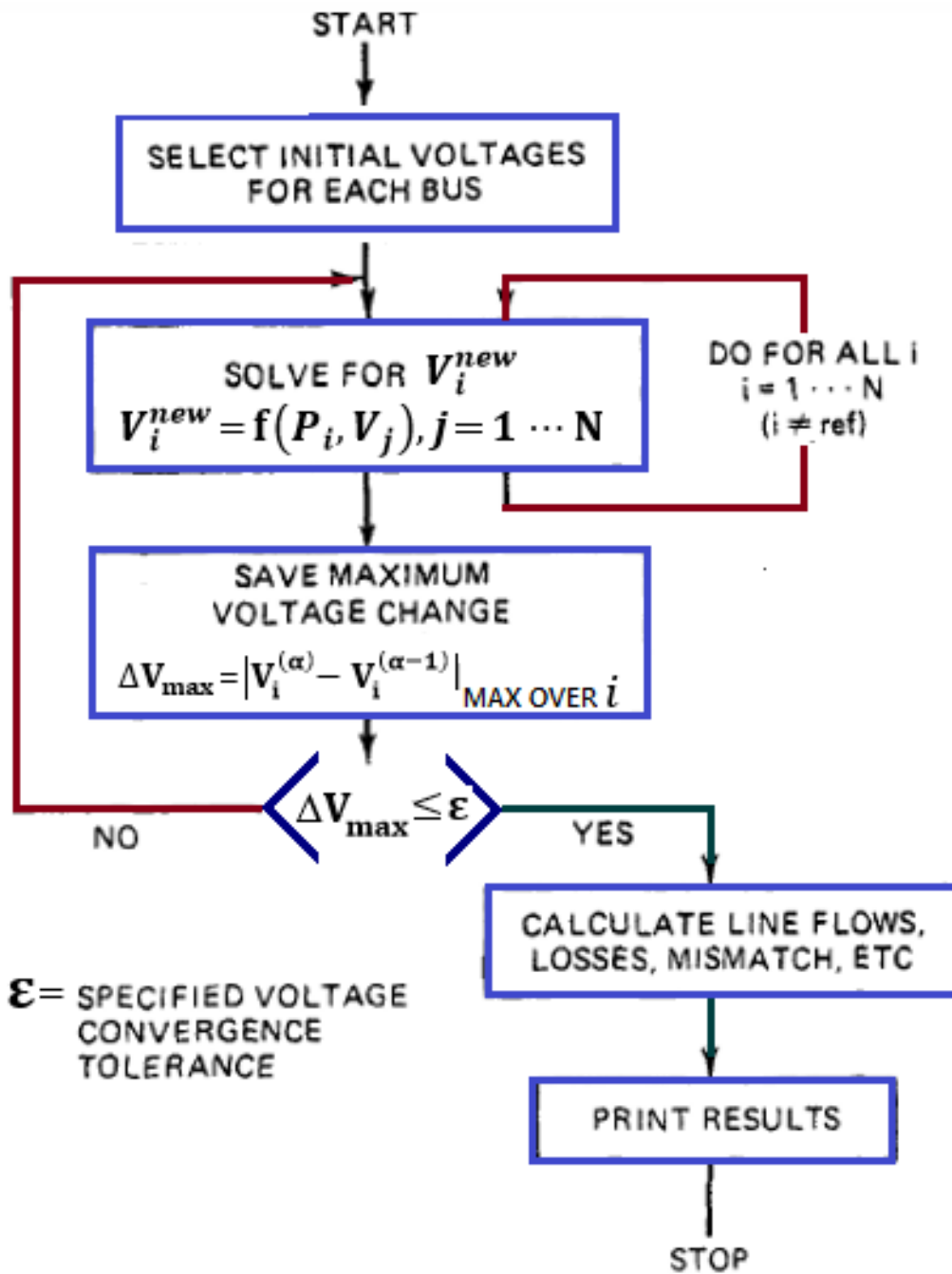


FIG. 1-3 Gauss-Seidel power-flow solution

<p><b>First iteration:</b></p> $V_2^{(0)} = V_3^{(0)} = 1.0$ $V_2^{(1)} = \frac{1}{9} \left( \frac{1.2}{1.0} + 4 + 5 \right) = 1.133$ $V_3^{(1)} = \frac{1}{15} \left( \frac{-1.5}{1.0} + 10 + 5(1.133) \right) = 0.944$ <p><math>\Delta V_{max} = 0.133</math> too large</p>	(1-8)
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Note: In calculating  $V_3^{(1)}$  we used the new value of  $V_2$  found in the first correction.

<p><b>Second iteration:</b></p> $V_2^{(2)} = \frac{1}{9} \left[ \frac{1.2}{1.133} + 4 + 5(0.944) \right] = 1.078$ $V_3^{(2)} = \frac{1}{15} \left[ \frac{-1.5}{0.944} + 10 + 5(1.078) \right] = 0.923$ $\Delta V_{max} =  1.133 - 1.087  = 0.046$ <p>And so forth until <math>\Delta V_{max} \leq \epsilon</math></p>	(1-9)
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### 1.2 The Formulation of the AC Power Flow

AC power flows involve several types of bus specifications, as shown in Figure 1-4. Note that  $[P, \delta]$ ,  $[Q, |V|]$ , and  $[Q, \delta]$  combinations are generally not used.

Bus Type	P	Q	V	$\delta$	Comments
Load	✓	✓			Usual Load representation
Voltage Controlled	✓		✓		Assume  V  is held constant no matter what Q is
Generator or Synchronous Condenser	✓		✓ when $Q^- < Q_g < Q^+$		Generator or synchronous condenser (P=0) has VAR limits
	✓	✓ when $Q_g < Q^-$ $Q_g > Q^+$			$Q^-$ minimum VAR limit $Q^+$ maximum VAR limit  V  is held as long as long as $Q_g$ is within limit
Fixed Z to Ground					Only Z is given
Reference			✓	✓	"Swing bus" must adjust net power to hold voltage constant (essential for solution)

FIG. 1-4 Power-flow bus specifications (quantities checked are the bus boundary conditions).

The transmission network consists of complex impedances between buses and from the buses to ground. An example is given in Figure 1-5.

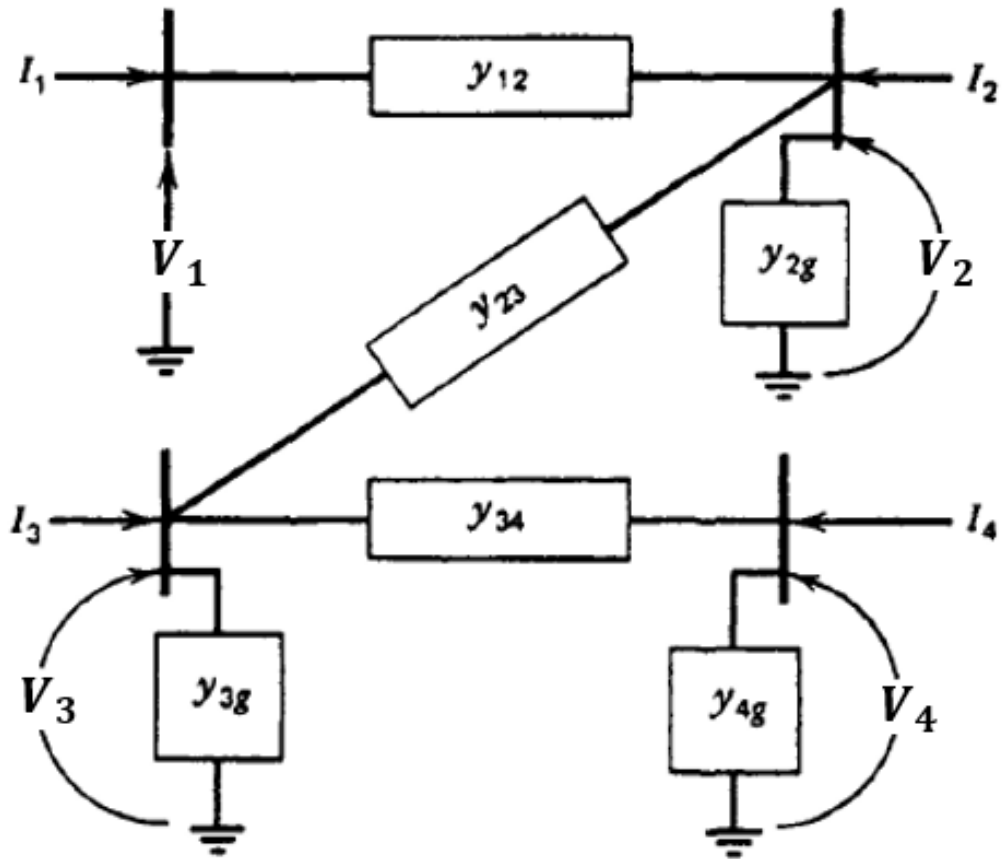


FIG. 1-5 Four-bus AC network.

The equations are written in matrix form as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_{12} & -y_{12} & 0 & 0 \\ -y_{12} & (y_{12} + y_{2g} + y_{23}) & -y_{23} & 0 \\ 0 & -y_{23} & (y_{23} + y_{3g} + y_{34}) & -y_{34} \\ 0 & 0 & -y_{34} & (y_{34} + y_{4g}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad (1-10)$$

(All  $I^s$ ,  $V^s$ ,  $y^s$  complex)

This matrix is called the network Y matrix, which is written as

$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & \mathbf{Y}_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & \mathbf{Y}_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & \mathbf{Y}_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$	(1-11)
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The rules for forming a  $\mathbf{Y}$  matrix are

If a line exists from  $i$  to  $j$

$$Y_{ij} = -y_{ij}$$

and

$$Y_{ii} = \sum_j y_{ij} + y_{ig}$$

$j$  overall lines connected to  $i$

The equation of net power injection at a bus is usually written as

$\frac{P_i - jQ_i}{V_i^*} = \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij}V_j + Y_{ii}V_i$	(1-12)
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